



## Introduction

- Any decision-making process involves choosing one of the existing alternatives. Each alternative can be characterized by a number of indicators, the expert assessment of the importance of which significantly affects the final choice
- It is necessary to conduct preparatory analytical work before moving on to direct comparison and choosing the best alternative:
  - form a series of indicators by which a comparison will be made;
  - evaluate the importance of the indicators relative to each other, that is, to obtain their numerical weight estimate.

**The problem of choosing an effective method for determining the significance of indicators that characterize the considered alternatives, and for obtaining numerical estimates of qualitative parameters, is still relevant today.**  
**When complex problems need to be solved, highly qualified experts with specific knowledge are involved in conducting a comparative analysis.**

## Hierarchy analysis method

All indicators are compared by an expert in pairs - The results form a matrix of paired comparisons

$$\begin{vmatrix} 1 & a_{12} & a_{12} \cdot a_{23} \\ \frac{1}{a_{12}} & 1 & a_{23} \\ \frac{1}{a_{12} \cdot a_{23}} & \frac{1}{a_{23}} & 1 \end{vmatrix}$$

However, the data obtained by experts are not completely consistent. T. Saati introduces a consistency index to process this matrix

$$C. I. = \frac{\lambda_{\max} - n}{n - 1}$$

The ratio of the C.I. to the average random index for a matrix of the same order (R.I.) is called the consistency ratio (C.R.).

When the matrix with expert values is obtained, it is necessary to calculate the C.R. and, if the C.R. exceeds 0.10, it is recommended that the judgment matrix be adjusted. Judgments are revised according to the following algorithm:

- Finding a row  $i$  such that  $\max_{j=1}^n \left| a_{ij} - \frac{w_i}{w_j} \right|$
- Replacement in this row and the corresponding column of all  $a_{ij}$  with  $\frac{w_i}{w_j}$
- Recalculation of the vector of the indicators' weights and C.R. value
- Repeat steps 1-3 if the C.R. is still above normal

## Visualization. Analysis of the paired comparison matrix

It is possible to build diagrams that would show an expert a visual picture of the comparisons. This is essential to make it easier for the expert to work with a large number of indicators – **Pie chart**  
To represent the T. Saati scale as a percentage, it is necessary to calculate the percentage step

$$h = \frac{y_{\max} - y_{\min}}{k}$$

Two approaches can be used to define  $y_{\max} - y_{\min}$ :

- $y_{\max} = 100\%$   
 $y_{\min} = 0\%$   
 $h = \frac{100\% - 0\%}{16} = 6,25\%$
- The expert is required to indicate the most important and most unimportant indicators and give them a 100-point rating.  
For example, the expert specified the following values:  $y_{\max} = 90$ , and  $y_{\min} = 10$ .  
 $h = \frac{90\% - 10\%}{16} = 5\%$

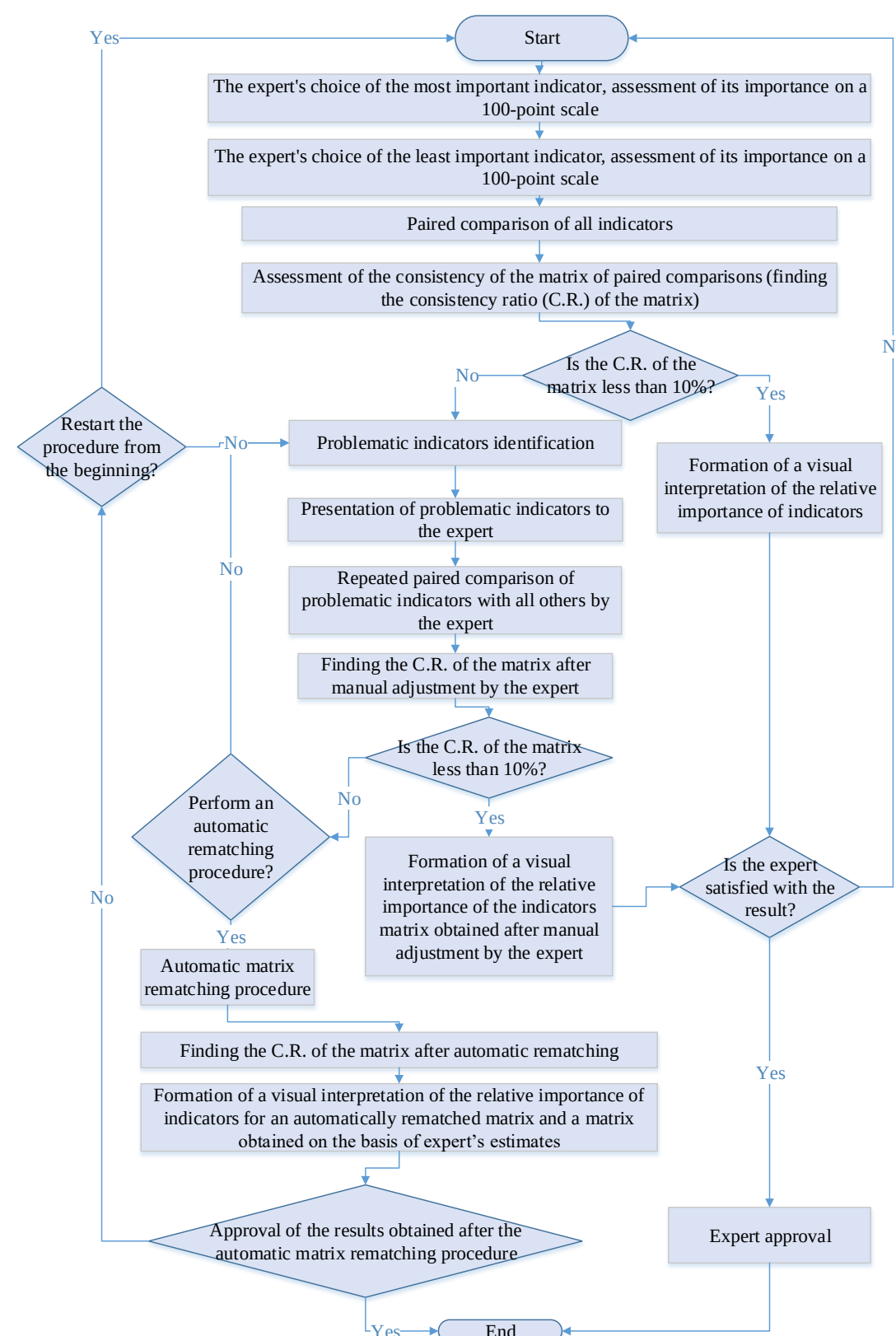
## Method implementation algorithm

### Key steps:

- the expert's choice of the most important indicator, assessment of its importance on a 100-point scale;
- the expert's choice of the least important indicator, assessment of its importance on a 100-point scale;
- paired comparison of all indicators;
- assessment of the consistency of the matrix of paired comparisons (finding the C.R.).

**If the C.R. is less than 10%, then further steps are taken only if the expert wishes:**

- identify problematic indicators;
- allow the expert to reassess the problematic indicators manu-ally;
- find the C.R. of the corrected matrix;
- run the automatic matrix rematching procedure;
- find the C.R. of the rematched matrix;
- generate a visual interpretation of the relative importance of the indicators for both matrices.

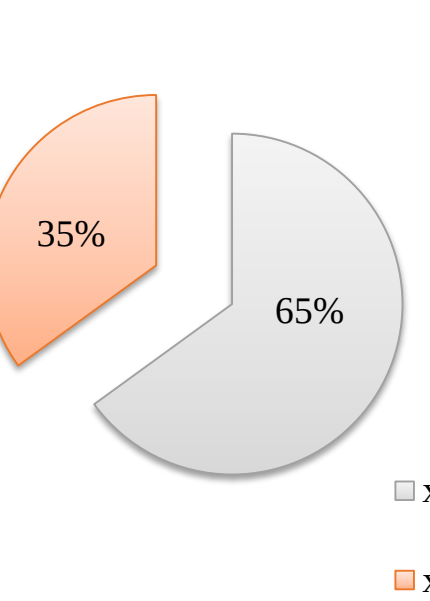


## Example of the proposed method application

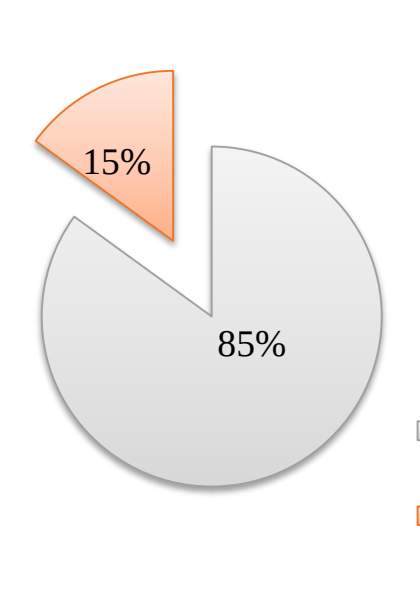
| Expert's estimates  |                |                |                |                |                |                |                |                |                |
|---------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Compared indicators | x <sub>1</sub> | x <sub>2</sub> | x <sub>3</sub> | x <sub>4</sub> | x <sub>5</sub> | x <sub>6</sub> | x <sub>7</sub> | x <sub>8</sub> | x <sub>9</sub> |
| x <sub>1</sub>      | 1              | 4              | 3              | 1              | 1/3            | 1/2            | 6              | 3              | 6              |
| x <sub>2</sub>      | 1/4            | 1              | 1              | 1/2            | 1/2            | 1/2            | 4              | 3              | 4              |
| x <sub>3</sub>      | 1/3            | 1              | 1              | 1/2            | 1/3            | 1/4            | 7              | 6              | 5              |
| x <sub>4</sub>      | 1              | 2              | 2              | 1              | 1/3            | 1/4            | 5              | 4              | 5              |
| x <sub>5</sub>      | 3              | 2              | 3              | 3              | 1              | 1              | 8              | 5              | 6              |
| x <sub>6</sub>      | 2              | 2              | 4              | 4              | 1              | 1              | 8              | 6              | 1/2            |
| x <sub>7</sub>      | 1/6            | 1/4            | 1/7            | 1/5            | 1/8            | 1/8            | 1              | 2              | 3              |
| x <sub>8</sub>      | 1/3            | 1/3            | 1/6            | 1/4            | 1/5            | 1/6            | 1/2            | 1              | 4              |
| x <sub>9</sub>      | 1/6            | 1/4            | 1/5            | 1/5            | 1/6            | 2              | 1/3            | 1/4            | 1              |

### Paired comparisons visualization

The importance ratio of indicators "x<sub>1</sub>" and "x<sub>2</sub>"



The importance ratio of indicators "x<sub>6</sub>" and "x<sub>7</sub>"



When all paired comparisons are made by an expert, we get a percentage ratio of all indicators. These values are displayed on a pie-chart by looking at which an expert determines whether he put the correct rating according to the proposed method. If it does not look balanced, he can immediately correct his estimates.

| Compared indicators | Relative importance (weight) |  |  |  |  |  |  |  |  |
|---------------------|------------------------------|--|--|--|--|--|--|--|--|
| x <sub>1</sub>      | 0,147                        |  |  |  |  |  |  |  |  |
| x <sub>2</sub>      | 0,085                        |  |  |  |  |  |  |  |  |
| x <sub>3</sub>      | 0,091                        |  |  |  |  |  |  |  |  |
| x <sub>4</sub>      | 0,119                        |  |  |  |  |  |  |  |  |
| x <sub>5</sub>      | 0,231                        |  |  |  |  |  |  |  |  |
| x <sub>6</sub>      | 0,245                        |  |  |  |  |  |  |  |  |
| x <sub>7</sub>      | 0,029                        |  |  |  |  |  |  |  |  |
| x <sub>8</sub>      | 0,033                        |  |  |  |  |  |  |  |  |
| x <sub>9</sub>      | 0,020                        |  |  |  |  |  |  |  |  |

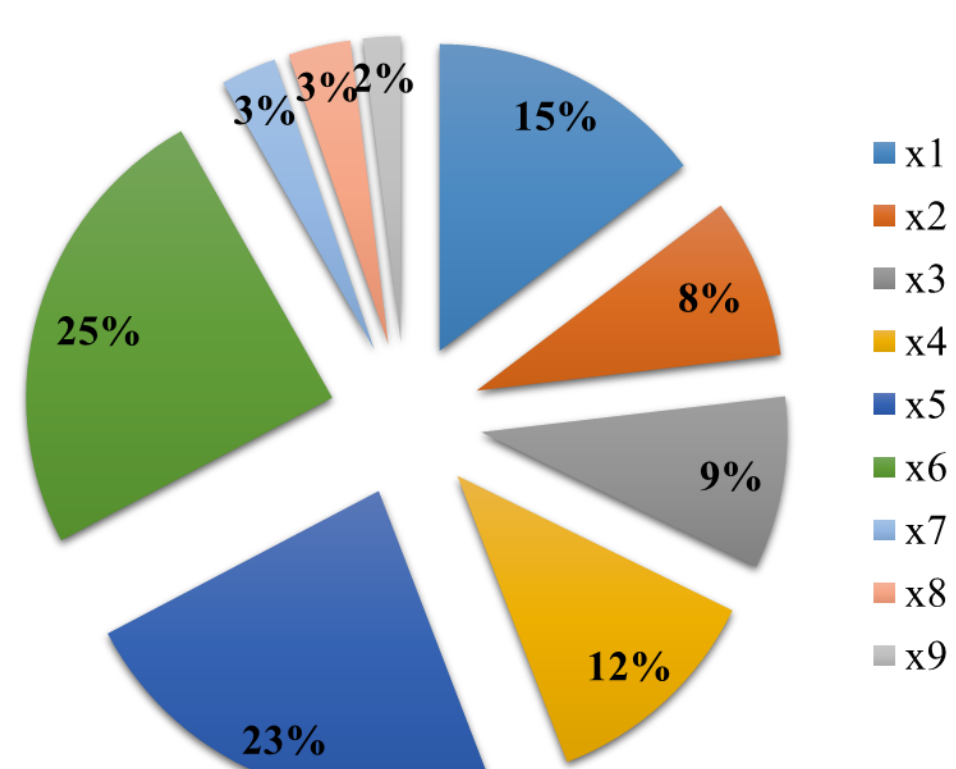
Weights that reflect the relative importance of all nine indicators were calculated based on the obtained data.  
**The C.R. of the matrix based on expert's estimates is 20.5%.**

| Compared indicators | x <sub>1</sub> | x <sub>2</sub> | x <sub>3</sub> | x <sub>4</sub> | x <sub>5</sub> | x <sub>6</sub> | x <sub>7</sub> | x <sub>8</sub> | x <sub>9</sub> |
|---------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| x <sub>1</sub>      | -              | 65%            | 60%            | 50%            | 40%            | 45%            | 75%            | 60%            | 75%            |
| x <sub>2</sub>      | 35%            | -              | 50%            | 45%            | 45%            | 45%            | 65%            | 60%            | 65%            |
| x <sub>3</sub>      | 40%            | 50%            | -              | 45%            | 40%            | 35%            | 80%            | 75%            | 70%            |
| x <sub>4</sub>      | 50%            | 55%            | 55%            | -              | 40%            | 35%            | 70%            | 65%            | 70%            |
| x <sub>5</sub>      | 55%            | 55%            | 60%            | 60%            | -              | 50%            | 85%            | 70%            | 75%            |
| x <sub>6</sub>      | 55%            | 55%            | 65%            | 65%            | 50%            | -              | 85%            | 75%            | 80%            |
| x <sub>7</sub>      | 25%            | 35%            | 20%            | 30%            | 15%            | 15%            | -              | 55%            | 60%            |
| x <sub>8</sub>      | 40%            | 40%            | 25%            | 35%            | 30%            | 25%            | 45%            | -              | 65%            |
| x <sub>9</sub>      | 25%            | 35%            | 30%            | 30%            | 25%            | 20%            | 40%            | 35%            | -              |

**The value of C.R. indicates that the matrix is not consistent,** and judgments require adjustment. So we obtain a new matrix in which percentages between pairs of indicators are represented

## Example of the proposed method application

### Percentages of compared indicators



### Weight estimates and rank of the indicators after manual correction of the matrix and automatic rematching of the matrix

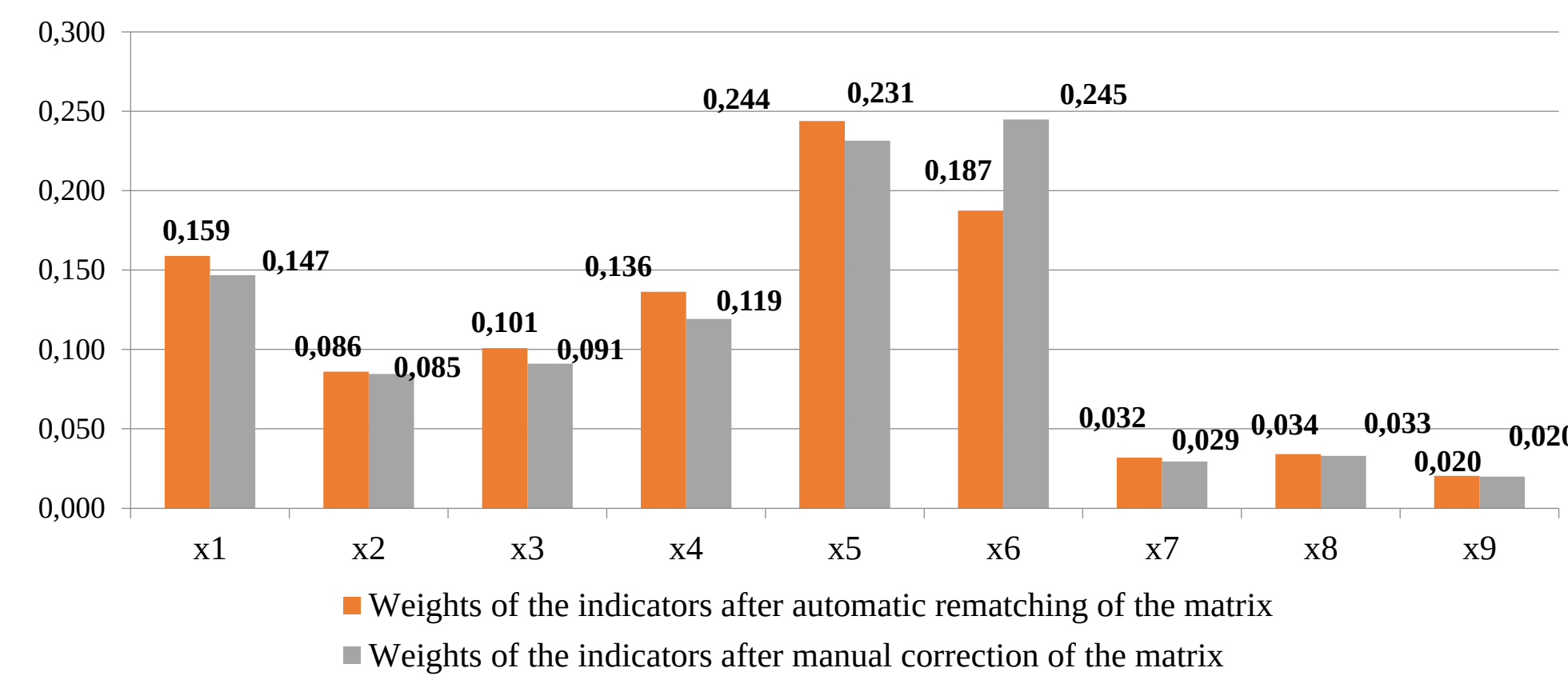
| X  | Weights after automatic rematching of the matrix | Rank X | Weights after manual correction by an expert | Rank X |
|----|--------------------------------------------------|--------|----------------------------------------------|--------|
| x1 | 0,159                                            | 3      | 0,147                                        | 3      |
| x2 | 0,086                                            | 6      | 0,085                                        | 6      |
| x3 | 0,101                                            | 5      | 0,091                                        | 5      |
| x4 | 0,136                                            | 4      | 0,119                                        | 4      |
| x5 | 0,244                                            | 1      | 0,231                                        | 2      |
| x6 | 0,187                                            | 2      | 0,245                                        | 1      |
| x7 | 0,032                                            | 8      | 0,029                                        | 8      |
| x8 | 0,034                                            | 7      | 0,033                                        | 7      |
| x9 | 0,020                                            | 9      | 0,020                                        | 9      |

Calculated weights are presented on the pie-chart. Expert can once again make sure that the technique he used is correct and the resulting values correspond to his opinion

We can see the difference in the absolute values of weight estimates, as well as in the ranks of indicators from the table

## Example of the proposed method application

### The difference in the weight estimates of the indicators after manual correction of the matrix and automatic rematching of the matrix



**This figure clearly shows the difference between the estimated weights of the indicators after manual correction and those after automatic rematching.** After the automatic rematching the leading indicator is x<sub>6</sub> but after the expert makes manual corrections, the leading indicator is x<sub>5</sub>.

## Conclusion

**The proposed algorithm** gives the expert a variety of opportunities to improve the accuracy of weights estimates using the T. Saati method. These opportunities include:

- prompt, demonstrative visualization of expert's estimates;
- identifying the indicators that made the greatest contribution to the inconsistency of the matrices and suggesting correcting them to the expert;
- the ability to choose between manual correction and automatic rematching of the matrix.